Received: *October 20, 1998,* Accepted: *October 7, 1998*

$SL(2,Z)$ Self-duality of super D3-brane action on $AdS_5 \times S^5$

Ichiro Oda

Edogawa University, 474 Komaki,
Nagareyama City, Chiba 270-0198, Japan
E-mail: ioda@edogawa-u.ac.jp

Abstract: It is shown that a supersymmetric and κ-symmetric D3-brane action on $AdS_5 \times S^5$ is mapped into itself by a duality transformaion, thereby verifying the $SL(2,Z)$ invariance of the D3-brane action in the $AdS_5 \times S^5$ background as in the flat background. To this end, we fix the κ-symmetry in a gauge which simplifies the classical action in order to perform an $SO(2)$ rotation of the $N = 2$ spinor index in a manifest way, though this may not be necessary. This situation is the same as the case of a super D-string on $AdS_5 \times S^5$ where it was shown that the super D-string action is transformed to a form of the IIB Green-Schwarz superstring action with the $SL(2,Z)$ covariant tension in the $AdS_5 \times S^5$ background through a duality transformation. These results strongly suggest that various duality relations originally found in the flat background may be independent of background geometry, in other words, the duality transformations in string and p-brane theories may exist even in general curved space-time.

Keywords: D-branes, String Duality.

JHEP10(1998)015

Contents

1. Introduction **1**

2. Super D3-brane action on $AdS_5 \times S^5$ **2**

3. $SL(2,Z)$ self-duality **5**

4. Discussions **9**

1. Introduction

Recently, studies of string and p-branes in the $AdS_5 \times S^5$ background have attracted a great deal of attention. These studies have been triggered by an interesting conjecture that the four dimensional large N super Yang-Mills theory is in a sense dual to the type IIB superstring theory on $AdS_5 \times S^5$ [1]. To fully understand this conjecture it is of great importance to understand various properties of string and p-brane theories in this background in more detail. In particular, $AdS_5 \times S^5$ is maximally supersymmetric 'vacuum' with 32 supercharges in addition to the flat ten dimensional space-time so superstring theory on this background would give rise to useful informations about the Kaluza-Klein compactification, quantum corrections through the no-go theorems from supersymmetry and a background independent formulation of the matrix model [2] etc.

A first step towards the construction of supersymmetric string and D3-brane actions on $AdS_5 \times S^5$ was taken by Metsaev and Tseytlin [3, 4], whose approach is based on the group manifold method [5]. Their fundamental strategy is to start with the supergroup $SU(2,2|4)$ directly, consider the coset superspace $SU(2,2|4)/(SO(4,1) \otimes SO(5))$, solve the Maurer-Cartan equations implied by the $su(2,2|4)$ superalgebra [6], and then find the supersymmetric and κ-symmetric action. Later, the gauge fixing of the κ-symmetry of the action was considered in terms of two different approaches [7, 8, 9, 10, 11], and the obtained actions were shown to be actually equivalent to each other [12].

Based on these studies, we have recently investigated the $SL(2,Z)$ S-duality [13] of type IIB superstring theory on $AdS_5 \times S^5$, where we have succeeded in proving that the super D-string action is also transformed to the type IIB Green-Schwarz superstring action [14] with the $SL(2,Z)$ covariant tension in the $AdS_5 \times S^5$ background in a quantum-mechanically exact manner as in the case of the flat Minkowskian background [15]. To address this issue one has taken account of the gauge fixing of the

κ-symmetry since in the classical action there are terms of higher orders in the spinor variables Θ reflecting the curved nature of the background metric, which seems to make it difficult to carry out the $SO(2)$ rotation in the duality transformation. This is to be contrasted with the situation in the flat background where only simple bilinear forms of the spinor variables appear in the classical action so that we do not have to fix the gauge symmetries at all. Here it is worth noting that the gauge fixing procedure does not change the physical contents of a theory so taking the proper gauge conditions is nothing but a recipe for understanding the duality transformation in a manifest way.

In this article, we wish to consider an extension of our previous studies of a super D-string action on $AdS_5 \times S^5$ [15] to a super D3-brane action on the same background [4]. It is now well-known that the super D3-brane action in the flat background [16, 17, 18, 19] is mapped into an equivalent D3-brane action by the duality transformation, in this sense, the super D3-brane is self-dual under $SL(2,Z)$ S-duality [18], so that it is natural to ask whether this peculiar feature of the super D3-brane on the flat background is also inherited to the case of the $AdS_5 \times S^5$ background or not. We will see that this is indeed the case by using a classical approach. The validity of the $SL(2,Z)$ duality of string and D3-brane theories in the $AdS_5 \times S^5$ background may lead to a strong expectation that various duality transformations found originally in the flat background would hold true even in an arbitrary curved background. We think that this observation is quite important for future developments of superstring theory and M-theory since these theories should be essentially formulated on the basis of not the flat Minkowskian but the general curved background in a background independent manner [2].

The contents of this article are organized as follows. In section 2 we shall review the super D3-brane action in the $AdS_5 \times S^5$ background which was constructed in [4]. In section 3 it will be shown that the super D3-brane action on this background is actually self-dual under the $SL(2,Z)$ duality transformation in terms of a classical method. To accomplish an appropriate $SO(2)$ rotation in an obvious way will require the gauge fixing of the κ-symmetry. The final section will be devoted to discussions, particularly a summary of the work at hand, the validity of the classical approximation and future works.

2. Super D3-brane action on $AdS_5 \times S^5$

We start by not only reviewing the studies of a super D3-brane action on $AdS_5 \times S^5$ [4] but also exposing a technical detail of the formalism to some extent. The κ-symmetric and reparametrization invariant super D3-brane action in the $AdS_5 \times S^5$ background constructed by Metsaev and Tseytlin [4] is given by

$$S = S_{DBI} + S_{WZ}\,, \tag{2.1}$$

with

$$\begin{aligned}
S_{DBI} &= -\int_{M_4} d^4\sigma\sqrt{-\det(G_{ij}+\mathcal{F}_{ij})}\,, \\
S_{WZ} &= \int_{M_5} H_5 = \int_{M_4=\partial M_5} \Omega_4\,, \\
H_5 &= d\Omega_4\,,
\end{aligned} \tag{2.2}$$

where the 3-brane tension is chosen to be a unity and the 5-form H_5 in the Wess-Zumino term is given by

$$\begin{aligned}
H_5 = {}& i\bar{L}\wedge(\frac{1}{6}\hat{L}\wedge\hat{L}\wedge\hat{L}\mathcal{E}+\mathcal{F}\wedge\hat{L}\mathcal{I})\wedge L \\
&+\frac{1}{30}\epsilon^{a_1\cdots a_5}L^{a_1}\wedge\cdots\wedge L^{a_5}+\frac{1}{30}\epsilon^{a'_1\cdots a'_5}L^{a'_1}\wedge\cdots\wedge L^{a'_5}\,,
\end{aligned} \tag{2.3}$$

where i and j run over the world-volume indices 0, 1, 2 and 3, and we have defined

$$\begin{aligned}
&G_{ij} = L_i^{\hat{a}}L_j^{\hat{a}},\ \ \hat{L}\equiv L^{\hat{a}}\Gamma^{\hat{a}}\,, \\
&\mathcal{E} = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix},\ \mathcal{I}=\sigma_1=\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix},\ \mathcal{K}=\sigma_3=\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}.
\end{aligned} \tag{2.4}$$

Here L^I and $L^{\hat{a}}$ are the Cartan 1-form spinor superfield and the vector superfield, respectively. They satisfy the Maurer-Cartan equations for $su(2,2|4)$ superalgebra [3, 4]

$$\begin{aligned}
dL^{\hat{a}} &= -L^{\hat{a}\hat{b}}\wedge L^{\hat{b}} - i\bar{L}\Gamma^{\hat{a}}\wedge L\,, \\
dL &= \frac{i}{2}\sigma_+\hat{L}\wedge\mathcal{E}L - \frac{1}{4}L^{\hat{a}\hat{b}}\Gamma^{\hat{a}\hat{b}}\wedge L, \\
d\bar{L} &= \frac{i}{2}\bar{L}\mathcal{E}\wedge\hat{L}\sigma_+ - \frac{1}{4}\bar{L}\Gamma^{\hat{a}\hat{b}}\wedge L^{\hat{a}\hat{b}}\,,
\end{aligned} \tag{2.5}$$

and σ_i are the Pauli matrices, which operate on $N=2$ spinor indices $I, J = 1, 2$. Throughout this article we follow the conventions and notations of references [3, 4, 11].

In the explicit parametrization $G(X,\Theta)=g(X)e^{\Theta Q}$ where $X^{\hat{m}}$, Θ^I, and Q_I are respectively the bosonic and fermionic space-time coordinates and 32-component supercharges [3], the action (2.1) takes the form

$$\begin{aligned}
S = {}& -\int_{M_4} d^4\sigma\sqrt{-\det(G_{ij}+\mathcal{F}_{ij})} + 2i\int_{M_4}\int_0^1 ds\ \bar{\Theta}(\frac{1}{6}\hat{L}_s\wedge\hat{L}_s\wedge\hat{L}_s\mathcal{E}+\mathcal{F}_s\wedge\hat{L}_s\mathcal{I})\wedge L_s \\
&+\int_{M_5} F_5\,,
\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}
\mathcal{F} &= F + 2i\int_0^1 ds\bar{\Theta}\hat{L}_s\wedge\mathcal{K}L_s\,, \\
\mathcal{F}_s &= F + 2i\int_0^s ds'\bar{\Theta}\hat{L}_{s'}\wedge\mathcal{K}L_{s'}\,, \\
F_5 &= \frac{1}{30}(\epsilon^{a_1\cdots a_5}e^{a_1}\wedge\cdots\wedge e^{a_5}+\epsilon^{a'_1\cdots a'_5}e^{a'_1}\wedge\cdots\wedge e^{a'_5})\,,
\end{aligned} \tag{2.7}$$

with $F = dA$ being the $U(1)$ gauge potential and $L^{\hat{a}}|_{\Theta=0} \equiv e^{\hat{a}}$ (the vielbein 1-form). We have used the standard rescaling trick $\Theta \to s\Theta$ and the Maurer-Cartan equations in deriving the second and third terms in the right-hand side of Eq.(2.6). Note that the two terms in $\int_{M_5} F_5$ describes that the self-dual RR 5-form field strength takes nontrivial values on AdS_5 and S^5, respectively. (Here we adopt the convention that the radii of AdS_5 and S^5 are a unity.) Actually this coupling precisely corresponds to the Freund-Rubin spontaneous compactification mechanism [20] in ten dimensions. It is surprising that the requirement of supersymmetry and κ-symmetry naturally leads to this coupling. The Cartan invariant 1-forms $L^I = L^I_{s=1}$ and $L^{\hat{a}} = L^{\hat{a}}_{s=1}$ are determined by solving the Maurer-Cartan equations (2.5) as follows [6]

$$\begin{aligned} L^I_s &= \left(\frac{\sinh(s\mathcal{M})}{\mathcal{M}} D\Theta \right)^I , \\ L^{\hat{a}}_s &= e^{\hat{a}}_{\hat{m}}(X) dX^{\hat{m}} - 4i\bar{\Theta}^I \Gamma^{\hat{a}} \left(\frac{\sinh^2(\frac{1}{2}s\mathcal{M})}{\mathcal{M}^2} D\Theta \right)^I , \end{aligned} \tag{2.8}$$

with

$$\begin{aligned} (\mathcal{M}^2)^{IL} &= \epsilon^{IJ}(-\gamma^a\Theta^J\bar{\Theta}^L\gamma^a + \gamma^{a'}\Theta^J\bar{\Theta}^L\gamma^{a'}) + \frac{1}{2}\epsilon^{KL}(\gamma^{ab}\Theta^I\bar{\Theta}^K\gamma^{ab} - \gamma^{a'b'}\Theta^I\bar{\Theta}^K\gamma^{a'b'}) , \\ (D\Theta)^I &= \left[d + \frac{1}{4}(\omega^{ab}\gamma_{ab} + \omega^{a'b'}\gamma_{a'b'}) \right] \Theta^I - \frac{1}{2} i\epsilon^{IJ}(e^a\gamma_a + ie^{a'}\gamma_{a'})\Theta^J . \end{aligned} \tag{2.9}$$

Now let us present the κ-transformation of the super D3-brane action (2.1) whose concrete expressions are given by [4]

$$\delta_\kappa \Theta^I = \kappa^I , \tag{2.10}$$

and the projection Γ is [16]

$$\begin{aligned} &\Gamma\kappa = \kappa, \ \Gamma^2 = 1, \ Tr\Gamma = 0 , \\ &\Gamma = \frac{\epsilon^{i_1\cdots i_4}}{\sqrt{-\det(G_{ij} + \mathcal{F}_{ij})}} \left(\frac{1}{4!}\Gamma_{i_1\cdots i_4}\mathcal{E} - \frac{1}{4}\Gamma_{i_1 i_2}\mathcal{F}_{i_3 i_4}\mathcal{I} + \frac{1}{8}\mathcal{F}_{i_1 i_2}\mathcal{F}_{i_3 i_4}\mathcal{E}\right) , \end{aligned} \tag{2.11}$$

with the definition of $\Gamma_{i_1\cdots i_n} \equiv \hat{L}_{[i_1} \cdots \hat{L}_{i_n]}$. Moreover, various superfields must transform under the κ-transformation as follows [3, 4]

$$\begin{aligned} \delta_\kappa L^{\hat{a}} &= 2i\bar{L}\Gamma^{\hat{a}}\delta_\kappa\Theta , \\ \delta_\kappa L &= d\delta_\kappa\Theta - \frac{i}{2}\sigma_+\hat{L}\mathcal{E}\delta_\kappa\Theta + \frac{1}{4}L^{\hat{a}\hat{b}}\Gamma^{\hat{a}\hat{b}}\delta_\kappa\Theta , \\ \delta_\kappa \bar{L} &= d\delta_\kappa\bar{\Theta} + \frac{i}{2}\delta_\kappa\bar{\Theta}\mathcal{E}\hat{L}\sigma_+ - \frac{1}{4}\delta_\kappa\bar{\Theta}\Gamma^{\hat{a}\hat{b}}L^{\hat{a}\hat{b}} , \\ \delta_\kappa G_{ij} &= 2i(\bar{L}_i\hat{L}_j + \bar{L}_j\hat{L}_i)\delta_\kappa\Theta , \\ \delta_\kappa \mathcal{F}_{ij} &= 2i(\bar{L}_i\mathcal{K}\hat{L}_j - \bar{L}_j\mathcal{K}\hat{L}_i)\delta_\kappa\Theta . \end{aligned} \tag{2.12}$$

Then it is a little tedious but straightforward to derive

$$\delta_\kappa H_5 = d\Lambda_4\,, \tag{2.13}$$

where

$$\Lambda_4 = 2i\bar{L}\wedge(\frac{1}{6}\hat{L}\wedge\hat{L}\wedge\hat{L}\mathcal{E}+\mathcal{F}\wedge\hat{L}\mathcal{I})\delta_\kappa\Theta\,. \tag{2.14}$$

In order to prove the equation (2.13), we have made use of the Maurer-Cartan equations (2.5) and the following cyclic Fierz identity for fermionic spinors A, B, C, D [16]

$$(\bar{A}\Gamma^{\hat{a}}B)\cdot(\bar{C}\Gamma^{\hat{a}}D) = -\frac{1}{2}\left[(\bar{A}\Gamma^{\hat{a}}e_I D)\cdot(\bar{B}\Gamma^{\hat{a}}e_I C)-(\bar{A}\Gamma^{\hat{a}}e_I C)\cdot(\bar{B}\Gamma^{\hat{a}}e_I D)\right]\,, \tag{2.15}$$

where $e_I = \{1, \mathcal{E}, \mathcal{I}, \mathcal{K}\}$. Then after a little tedious calculations it turns out that the super D3-brane action on $AdS_5\times S^5$ is invariant under the κ transformation

$$\delta_{\Gamma\kappa}S_{DBI}+\delta_\kappa S_{WZ} = 0\,, \tag{2.16}$$

where we have used various matrix formulas and Γ matrix identity such as

$$\begin{aligned}
\delta\det M &= \det M\cdot Tr(M^{-1}\delta M)\,,\\
(G+\mathcal{F})^{-1ji} &= (G-\mathcal{F})^{-1ij}\,,\\
(1\pm G^{ik}\mathcal{F}_{kj})^{-1} &= (1+(G^{ik}\mathcal{F}_{kj})^2)^{-1}(1\mp G^{ik}\mathcal{F}_{kj})\,,\\
\Gamma^{\hat{a}}\Gamma^{\hat{a_1}\cdots\hat{a_{2n}}} &= \Gamma^{\hat{a}\hat{a}_1\cdots\hat{a}_{2n}}+2n\eta^{\hat{a}[\hat{a}_1}\Gamma^{\hat{a}_2\cdots\hat{a}_{2n}]}\,.
\end{aligned} \tag{2.17}$$

3. $SL(2,Z)$ self-duality

We now turn our attention to a proof of self-duality of the super D3-brane action under the $SL(2,Z)$ duality transformation. In the flat background this issue was studied in [21] for the bosonic case and in [18] for the supersymmetric case. One of the crucial issues left so far unsolved is to examine whether this self-duality is valid even in a curved background. In fact, in the final section in [18], it is stated that "..... For the most part, our analysis has been classical and limited to flat backgrounds. The results should not depend on these restrictions, however." Motivated by the sentences, we have recently clarified that a type IIB superstring action on $AdS_5\times S^5$ exhibits the $SL(2,Z)$ S-duality as in the case of the flat background [15]. In this section, we wish to show the expected $SL(2,Z)$ invariance of the super D3-brane reviewed in the previous section.

In the work of the super D-string on $AdS_5\times S^5$ [15] we have used the path integral based on the first-order Hamiltonian formalism [22, 23] since this method is effective in showing the equivalence at the quantum level. Instead, in the present article, we shall employ a classical approach developed in [21, 18]. The reason why we use the classical approach will be argued to some extent in the final section.

Let us consider the following action:

$$S = -\int_{M_4} d^4\sigma \sqrt{-\det(G_{ij} + \mathcal{F}_{ij})} + 2i \int_{M_4} \int_0^1 ds\ \bar{\Theta}(\frac{1}{6}\hat{L}_s \wedge \hat{L}_s \wedge \hat{L}_s \mathcal{E} + \mathcal{F}_s \wedge \hat{L}_s \mathcal{I}) \wedge L_s + \\ + \int_{M_5} F_5 + \int_{M_4} \frac{1}{2} C_0 F \wedge F \,, \tag{3.1}$$

where we have added the topological term $\frac{1}{2}C_0 F \wedge F$ with a constant axion C_0 to the action (2.6) for later convenience. However, this expression (3.1) is not so illuminating for the purpose of an analysis of the duality transformation. Thus, instead, we shall consider an alternative form of the action equivalent to (3.1), which is given by

$$S = -\int_{M_4} d^4\sigma \sqrt{-\det(G_{ij} + \mathcal{F}_{ij})} + \int_{M_4} (C_4 + C_2 \wedge \mathcal{F} + \frac{1}{2} C_0 F \wedge F) \,, \tag{3.2}$$

whose superficial expression is very similar to that adopted in [18], but of course the physical contents are different. However, we will see that this form of the action would be useful in the analysis of the self-duality of the super D3-brane action on $AdS_5 \times S^5$ since we can follow similar arguments to as in [18].

To make contact with (3.1) one has to express the 4-form field C_4 and 2-form field C_2 in terms of the superfields and the supercoordinates existing in the theory at hand. This is provided by the relation

$$H_5 = d\Omega_4 \,, \ \Omega_4 = C_4 + C_2 \wedge \mathcal{F} \,, \tag{3.3}$$

by which we have the coupled differential equations to be solved for C_4 and C_2

$$\begin{aligned} dC_4 - C_2 \wedge db_2 &= i\bar{L} \wedge \frac{1}{6}\hat{L} \wedge \hat{L} \wedge \hat{L}\mathcal{E} \wedge L \\ &\quad + \frac{1}{30}\epsilon^{a_1 \cdots a_5} L^{a_1} \wedge \cdots \wedge L^{a_5} + \frac{1}{30}\epsilon^{a'_1 \cdots a'_5} L^{a'_1} \wedge \cdots \wedge L^{a'_5} \,, \\ dC_2 \wedge \mathcal{F} &= i\bar{L} \wedge \mathcal{F} \wedge \hat{L}\mathcal{I} \wedge L \,, \end{aligned} \tag{3.4}$$

where we have defined as

$$\begin{aligned} \mathcal{F} &= F - b_2, \\ b_2 &= -2i \int_0^1 ds \bar{\Theta} \hat{L}_s \wedge \mathcal{K} L_s \,. \end{aligned} \tag{3.5}$$

Then it is relatively straightforward to solve the second equation in Eq.(3.4) whose result is given by

$$C_2 = 2i \int_0^1 ds \bar{\Theta} \hat{L}_s \wedge \mathcal{I} L_s \,, \tag{3.6}$$

where the Maurer-Cartan equations and the trick $\Theta \to s\Theta$ were utilized again. The first equation in Eq.(3.4) can be also solved with the help of Eq.(3.6), but it is unnecessary to use the concrete expression of C_4 for the present consideration so we omit to write it down explicitly.

We are now in a position to think of the duality transformation of the action (3.2). Following a similar path of derivation to the super D3-brane in the flat background [18], first of all, we have to add $\int d^4\sigma \frac{1}{2}\tilde{H}^{ij}(F_{ij} - 2\partial_i A_j)$ to the action (3.2), and then take the variation with respect to A_i, which gives us the solution $\tilde{H}^{ij} = \epsilon^{ijkl}\partial_k B_l$ where B_i is a vector field. After substituting this solution into the action (3.2) and solving the variational equation for F_{ij} to rewrite the action in terms of B_i instead of F_{ij}, we arrive at the dual action of (3.2)

$$S_D = -\int_{M_4} d^4\sigma \sqrt{-\det\left[G_{ij} + \frac{1}{\sqrt{1+C_0^2}}(\tilde{F}_{ij} + C_{ij} + C_0 b_{ij})\right]} + \int_{M_4} \Omega_D\,, \qquad (3.7)$$

where $\tilde{F} = dB$ and Ω_D is defined by

$$\begin{aligned} \Omega_D &= C_4 - b_2 \wedge C_2 - \frac{1}{2}C_0 b_2 \wedge b_2 + b_2 \wedge (\tilde{F} + C_2 + C_0 b_2) - \\ &- \frac{C_0}{2(1+C_0^2)}(\tilde{F} + C_2 + C_0 b_2) \wedge (\tilde{F} + C_2 + C_0 b_2)\,. \end{aligned} \qquad (3.8)$$

In order to prove the self-duality one has to show that S_D has the same form as the original action (3.2). To this aim, we perform an appropriate $SO(2)$ rotation of the spinor coordinates Θ^I, which amounts to the following $SO(2)$ rotation of the 'Pauli matrices'

$$\begin{aligned} \mathcal{I}' &= -\frac{1}{\sqrt{1+C_0^2}}(\mathcal{K} + C_0\mathcal{I})\,, \\ \mathcal{K}' &= \frac{1}{\sqrt{1+C_0^2}}(\mathcal{I} - C_0\mathcal{K})\,. \end{aligned} \qquad (3.9)$$

A delicate problem of how one performs the $SO(2)$ rotation of the spinor coordinates Θ^I will discussed below in detail. Then, we obtain

$$\frac{1}{\sqrt{1+C_0^2}}(\tilde{F} + C_2 + C_0 b_2) = \tilde{F}' - b_2' \equiv \tilde{\mathcal{F}}'\,, \qquad (3.10)$$

where we have defined $\tilde{F}' \equiv \frac{1}{\sqrt{1+C_0^2}}\tilde{F}$. Consequently, the Dirac-Born-Infeld term in S_D can be rewritten to be the same form as the original action. Moreover, from (3.4)-(3.6) and (3.8)-(3.10), we can evaluate the Wess-Zumino part as follows:

$$\begin{aligned} d\Omega_D &\equiv dC_4' - C_2' \wedge db_2' + dC_2' \wedge \tilde{\mathcal{F}}' \\ &= i\bar{L} \wedge (\frac{1}{6}\hat{L} \wedge \hat{L} \wedge \hat{L}\mathcal{E}' + \tilde{\mathcal{F}}' \wedge \hat{L}\mathcal{I}') \wedge L + \\ &+ \frac{1}{30}\epsilon^{a_1 \cdots a_5} L^{a_1} \wedge \cdots \wedge L^{a_5} + \frac{1}{30}\epsilon^{a_1' \cdots a_5'} L^{a_1'} \wedge \cdots \wedge L^{a_5'}\,, \end{aligned} \qquad (3.11)$$

where we have used the fact that $\mathcal{E}' = \mathcal{E}$ under the $SO(2)$ rotation (3.9). Accordingly, the dual action of the super D3-brane action on $AdS_5 \times S^5$ can be rewritten as

$$S = -\int_{M_4} d^4\sigma \sqrt{-\det(G_{ij} + \tilde{\mathcal{F}}'_{ij})} + \int_{M_4} (C'_4 + C'_2 \wedge \tilde{\mathcal{F}}' + \frac{1}{2} C_0 \tilde{F}' \wedge \tilde{F}') \,, \quad (3.12)$$

In this way, we can prove the self-duality of the super D3-brane action on $AdS_5 \times S^5$ in a similar way to the case of the flat background.

To understand the $SL(2,Z)$ duality more clearly, it is useful to introduce a constant dilaton background in addition to a constant axion background. Provided that one starts with the action [18]

$$S = -\int_{M_4} d^4\sigma \sqrt{-\det(G_{ij} + e^{-\frac{\phi}{2}} F_{ij} - b_{ij})} + \\ + \int_{M_4} \left(C_4 + C_2 \wedge (e^{-\frac{\phi}{2}} F - b_2) + \frac{1}{2} C_0 F \wedge F \right) , \quad (3.13)$$

one can show that this supersymmetric and κ-symmetric action is also self-dual with the desired $SL(2,Z)$ transformation law of the dilaton and the axion

$$e^{-\phi} \to \frac{1}{e^{-\phi} + e^{\phi} C_0^2}, \qquad C_0 \to -\frac{e^{\phi} C_0}{e^{-\phi} + e^{\phi} C_0^2} \,. \quad (3.14)$$

Before closing this section, we should discuss the $SO(2)$ rotation of the spinor coordinates, or equivalently, the $SO(2)$ rotation of the 'Pauli matrices' (3.9). As already studied in the duality transformation of a type IIB superstring on $AdS_5 \times S^5$ [15], at the present case we shall also take a convenient gauge condition of the κ-symmetry to make the fermionic structure of the classical action simpler. This is because owing to a complicated dependence on the spinor coordinates Θ of the classical action as seen in Eqs.(2.8) and (2.9) it is not always obvious that we can carry out the desired $SO(2)$ rotation in the classical action. Of course, we do not intend to exclude other possibilities of performing the $SO(2)$ rotation at all. For instance, as another attempt, it turns out to be advantageous to parametrize the supergroup element $G(X, \Theta)$ in a specific parametrization reflecting a underlying geometrical structure, by which the classical action would become a rather simpler form than the case at hand. Then there might exist some ingenious prescription for the $SO(2)$ rotation of the spinor coordinates even without fixing any local symmetries.

In this article, following the previous work [15] we shall utilize the recently developed quantization method [10, 11][1] where the gauge condition of the κ-symmetry was selected to be

$$\Theta^I_- = 0 \,, \quad (3.15)$$

[1]More recently, some subtleties associated with consistent gauge fixing world-volume diffeomorphisms and κ-symmetry of superbrane actions have been discussed [24]. These subtleties do not change our conclusion since we can also show that the desired $SO(2)$ rotation is sucessfully performed without fixing κ-symmetry, whose detail will be reported in a separate publication.

JHEP10(1998)015

where

$$\Theta_{\pm}^{I} \equiv \mathcal{P}_{\pm}^{IJ}\Theta^{J},\ \mathcal{P}_{\pm}^{IJ} \equiv \frac{1}{2}(\delta^{IJ} \pm \Gamma_{0123}\epsilon^{IJ})\,. \tag{3.16}$$

One of the remakable things is that in this gauge (3.15) the superfields take the simple forms given by

$$L_{is}^{t} = \frac{1}{y}\partial_i y^t,\ L_{js}^{I} = sy^{\frac{1}{2}}\partial_i\theta_{+}^{I}\,, \tag{3.17}$$

where $t = 4, \cdots, 9$, y is a coordinate of $AdS_5 \times S^5$ in the Cartesian coordinate, and $\Theta_{+}^{I} \equiv y^{\frac{1}{2}}\theta_{+}^{I}$ [10, 11].

If we introduce the orthogonal matrix U for the $SO(2)$ rotation of the spinor coordinates such that $\Theta^I = U^{IJ}\tilde{\Theta}^J$ where $U = \frac{1}{\sqrt{1+(C_0-\sqrt{1+C_0^2})^2}}[(-C_0 + \sqrt{1+C_0^2})1 - \mathcal{E}]$, it is easy to show that this matrix U indeed induces the $SO(2)$ rotation of the 'Pauli matrices' (3.9) since we have

$$U^T(\mathcal{I} - C_0\mathcal{K})U = \sqrt{1+C_0^2}\mathcal{K}\,. \tag{3.18}$$

As shown in the previous work [15], the projector $\mathcal{P}_{\pm}^{IJ}$ is invariant under the orthogonal transformation by the matrix U, which yields the important relation $\theta_{+}^{I} = U^{IJ}\tilde{\theta}_{+}^{J}$. Since in the gauge condition (3.15) the action of super D3-brane becomes a very simple form, we can carry out the transformation (3.18) with keeping the other irrelevant terms including the spinor coordinates unchanged. (It is useful to use the expression (3.1) rather than (3.2) of the action to prove this fact.) Hence we have shown that the $SO(2)$ rotation of the 'Pauli matrices' (3.9) is mathematically allowed if we fix the gauge associated with the κ-symmetry.

4. Discussions

In this paper, we have studied the property of the $SL(2,Z)$ self-duality of a supersymmetric and κ-symmetric D3-brane action in the $AdS_5 \times S^5$ background. We have seen that on this background the dilaton and the axion undergo the same the $SL(2,Z)$ duality transformation as in the case of the flat Minkowskian background. It is interesting to notice that although the physical contents of a theory in the two backgrounds are different, the derivation method of the $SL(2,Z)$ duality transformation is quite similar. We think that this similarity holds even in proving the self-duality of super D-brane theories in general background geometries.

In this article, we have confined ourselves to a classical analysis to prove the self-duality of the super D3-brane action. Of course, a quantum-mechanically exact analysis would be welcoming in future. However, this problem may not be so important by the following two reasons. One reason comes from the fact that the action of p-branes

($p \geq 2$) is in essense unrenormalizable so that new degrees of dynamical freedom and new higher order terms in general appear in approaching the short distance regime. In this sense, the exact treatment within the present context, i.e., showing the self-duality of classical action of super D3-brane in a quantum-mechanical way, is not so interesting. The other stems from a reasoning somewhat contrary to the first statement. Namely, the super D3-brane action on $AdS_5 \times S^5$ may be an exact conformal field theory and receive no quantum corrections except the renormalization of an overall factor owing to the no-go theorems from the maximal supersymmetry. Actually we have already had some articles discussing the related problem [25, 26]. Accordingly, from this vantage point the classical analysis presented in this article may be sufficient as an exact proof.

Finally we would like to make comments on future works. One interesting direction is to construct general super D-brane actions on $AdS_5 \times S^5$ and investigate various duality transformations along the present analysis. In paticular, as understood from an analysis in the flat background we know that super D2-brane and D4-brane actions would transform in a manner that is expected from the relation between type IIA superstring theory and 11 dimensional M theory. We believe that this behavior is also valid in the case of the $AdS_5 \times S^5$. We would like to clarify this point in near future.

Another interesting problem which is also related to the first work is to construct a M5-brane theory on $AdS_7 \times S^4$. The following strategy is perhaps useful, namely, start with the supergroup $OSp(6,2|4)$ directly, consider the coset superspace $OSp(6,2|4)/(SO(6,1) \otimes SO(4))$, solve the Maurer-Cartan equations implied by the $osp(6,2|4)$ superalgebra, and then find the supersymmetric and κ-symmetric action, which would become the form as the PST action [27] in the flat background limit except the Freund-Rubin coupling. We can conjecture that the D4-brane action would be also identical to the action by double-dimensional reduction of the so-obtained M5-brane action in the $AdS_7 \times S^4$ background as in the flat background, so studies of the super D4-brane action may in turn yield some useful informations in completing this program. (See the related papers [28, 29].) We also return to this problem in future.

Acknowledgments

This work was supported in part by Grant-Aid for Scientific Research from Ministry of Education, Science and Culture No.09740212.

References

[1] J. Maldacena, `hep-th/9711200`.

[2] I. Oda, *Mod. Phys. Lett.* **A 13** (1998) 203 [`hep-th/9709005`]; *Nucl. Phys.* **B 516** (1998) 160 [`hep-th/9710030`]; *Phys. Lett.* **B 427** (1998) 267 [`hep-th/9801051`]; `hep-th/9801085`; `hep-th/9806096`.

[3] R.R. Metsaev and A.A. Tseytlin, hep-th/9805028.

[4] R.R. Metsaev and A.A. Tseytlin, hep-th/9806095.

[5] L. Castellani, R.D. Auria and P. Fre, *Supergravity and Superstrings: A Geometric Perspective*. Vol 1,2,3, World Scientific, 1991.

[6] R. Kallosh, J. Rahmfeld and A. Rajaraman, *JHEP* **09** (1998) 002 [hep-th/9805217].

[7] G. Dall'Agata, D. Fabbri, C. Fraser, P. Fre, P. Termonia and M. Trigiante, hep-th/9807115.

[8] I. Pesando, *JHEP* **04** (1998) 016 [hep-th/9802020].

[9] R. Kallosh, hep-th/9807206.

[10] R. Kallosh and J. Rahmfeld, hep-th/9808038.

[11] R. Kallosh and A.A. Tseytlin, *JHEP* **10** (1998) 016 [hep-th/9808088].

[12] I. Pesando, hep-th/9808146.

[13] J.H. Schwarz, *Phys. Lett.* **B 360** (1995) 13 [hep-th/9508143]; Erratum, *ibid.* **364** (1995) 252.

[14] M.B. Green and J.H. Schwarz, *Phys. Lett.* **B 136** (1984) 367.

[15] I. Oda, hep-th/9809076.

[16] M. Cederwall, A. von Gussich, B.E.W. Nilsson and A. Westerberg, *Nucl. Phys.* **B 490** (1997) 163 [hep-th/9610148].

[17] J. Aganagic, C. Popescu and J.H. Schwarz, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249]; *Nucl. Phys.* **B 495** (1997) 99 [hep-th/9612080].

[18] J. Aganagic, J. Park, C. Popescu and J.H. Schwarz, *Nucl. Phys.* **B 496** (1997) 215 [hep-th/9702133].

[19] E. Bergshoeff and P.K. Townsend, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[20] P.G.O. Freund and M.A. Rubin, *Phys. Lett.* **B 97** (1980) 233.

[21] A.A. Tseytlin, *Nucl. Phys.* **B 469** (1996) 51 [hep-th/9602064].

[22] S.P. de Alwis and K. Sato, *Phys. Rev.* **D 53** (1996) 7187 [hep-th/9601167].

[23] I. Oda, *Phys. Lett.* **B 430** (1998) 242 [hep-th/9802152].

[24] P. Pasti, D. Sorokin and M. Tonin, hep-th/9809213.

[25] T. Banks and M.B. Green, *JHEP* **05** (1998) 002 [hep-th/9804170].

[26] R. Kallosh and A. Rajaraman, hep-th/9805041.

[27] P. Pasti, D. Sorokin and M. Tonin, *Phys. Lett.* **B 398** (1997) 41 [hep-th/9701037]; I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, *Phys. Rev. Lett.* **78** (1997) 4332 [hep-th/9701149]; *Phys. Lett.* **B 408** (1997) 135 [hep-th/9703127].

[28] B. de Witt, K. Peeters, J. Plefka and A. Sevrin, hep-th/9808052.

[29] P. Claus, hep-th/9809045.